

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4761

Mechanics 1

Friday

14 JANUARY 2005

Morning

1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- · You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g m s^{-2}$. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

2

Section A (36 marks)

1 The position vector, \mathbf{r} , of a particle of mass 4 kg at time t is given by

$$\mathbf{r} = t^2 \mathbf{i} + (5t - 2t^2) \mathbf{j},$$

where i and j are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle.

[4]

The particle is subject to a force F and a force 12 j N.

(ii) Find \mathbf{F} . [3]

2 Particles of mass 2 kg and 4 kg are attached to the ends X and Y of a light, inextensible string. The string passes round fixed, smooth pulleys at P, Q and R, as shown in Fig. 2. The system is released from rest with the string taut.

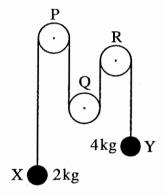


Fig. 2

- (i) State what information in the question tells you that
 - (A) the tension is the same throughout the string,
 - (B) the magnitudes of the accelerations of the particles at X and Y are the same. [2]

The tension in the string is TN and the magnitude of the acceleration of the particles is $a \,\mathrm{m\,s^{-2}}$.

- (ii) Draw a diagram showing the forces acting at X and a diagram showing the forces acting at Y.
- (iii) Write down equations of motion for the particles at X and at Y. Hence calculate the values of T and a. [5]

- 3 A particle is in equilibrium when acted on by the forces $\begin{pmatrix} x \\ -7 \\ z \end{pmatrix}$, $\begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$, where the units are newtons.
 - (i) Find the values of x, y and z. [4]
 - (ii) Calculate the magnitude of $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$. [2]
- 4 A particle is projected vertically upwards from a point O at 21 m s⁻¹.
 - (i) Calculate the greatest height reached by the particle.

When this particle is at its highest point, a second particle is projected vertically upwards from O at 15 ms⁻¹.

- (ii) Show that the particles collide 1.5 seconds later and determine the height above O at which the collision takes place. [6]
- A small box B of weight 400 N is held in equilibrium by two light strings AB and BC. The string BC is fixed at C. The end A of string AB is fixed so that AB is at an angle α to the vertical where $\alpha < 60^{\circ}$. String BC is at 60° to the vertical. This information is shown in Fig. 5.

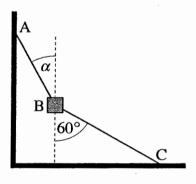


Fig. 5

(i) Draw a labelled diagram showing all the forces acting on the box.

[1]

[2]

(ii) In one situation string AB is fixed so that $\alpha = 30^{\circ}$.

By drawing a triangle of forces, or otherwise, calculate the tension in the string BC and the tension in the string AB. [4]

(iii) Show carefully, but briefly, that the box cannot be in equilibrium if $\alpha = 60^{\circ}$ and BC remains at 60° to the vertical. [2]

7 The trajectory ABCD of a small stone moving with negligible air resistance is shown in Fig. 7. AD is horizontal and BC is parallel to AD.

The stone is projected from A with speed $40 \,\mathrm{m\,s^{-1}}$ at 50° to the horizontal.

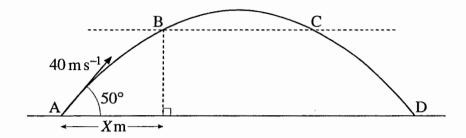


Fig. 7

- (i) Write down an expression for the horizontal displacement from A of the stone t seconds after projection. Write down also an expression for the vertical displacement at time t. [3]
- (ii) Show that the stone takes 6.253 seconds (to three decimal places) to travel from A to D. Calculate the range of the stone. [5]

You are given that X = 30.

- (iii) Calculate the time it takes the stone to reach B. Hence determine the time for it to travel from A to C. [4]
- (iv) Calculate the direction of the motion of the stone at C. [5]

Section B (36 marks)

6 In this question take g as $10 \,\mathrm{m \, s^{-2}}$.

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 6.

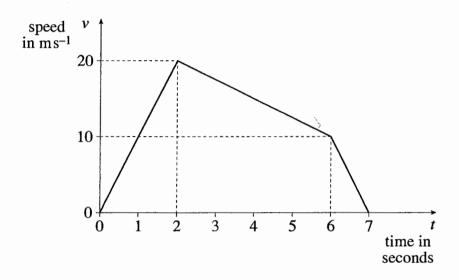


Fig. 6

For this model,

(i) calculate the distance fallen from
$$t = 0$$
 to $t = 7$, [3]

(ii) find the acceleration of the ball from
$$t = 2$$
 to $t = 6$, specifying the direction, [3]

(iii) obtain an expression in terms of
$$t$$
 for the downward speed of the ball from $t = 2$ to $t = 6$, [3]

(iv) state the assumption that has been made about the resistance to motion from
$$t = 0$$
 to $t = 2$. [1]

The part of the motion from t = 2 to t = 7 is now modelled by $v = -\frac{3}{2}t^2 + \frac{19}{2}t + 7$.

(v) Verify that v agrees with the values given in Fig. 6 at
$$t = 2$$
, $t = 6$ and $t = 7$. [2]

(vi) Calculate the distance fallen from
$$t = 2$$
 to $t = 7$ according to this model. [7]